THE MATHEMATICS TEACHER

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ASSOCIATED WITH

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EDITORIAL.

Teachers' Bureau.

The editors of this magazine are so often asked to recommend teachers for positions in various educational institutions, that it has been thought best to offer the use of the magazine to institutions which need teachers, and to teachers of mathematics who wish to obtain better positions. The service will be confidential, no names being published, and letters from teachers applying for positions will be forwarded to the school in which the position is open.

In order to make this service of use, it is necessary that teachers of mathematics bring it to the attention of the heads of their institutions, and that they themselves take advantage of it. The service will be free, unless the work assumes a magnitude that makes it necessary to charge a small fee for insertions.

The following notices are samples to indicate the general form to be used:

Schools and Colleges Needing Teachers.

Wanted: Woman teacher of secondary mathematics for high school in city of thirty thousand. Must have college degree and at least three years of secondary experience. Salary according to preparation and experience. Address S B 5.

Wanted: Man of experience in teaching college mathematics, for assistant professorship in small college in Pennsylvania.

Prefer one who knows conditions in secondary schools, and who is under thirty-five years of age. Address C A 17.

Teachers Desiring Positions.

Man, teacher of secondary mathematics, desires better position. Master of Arts, five years' experience, three in public, two in private school. Can be of value in other school activities. Least salary $1,600. Address K 5.

Directions.

Address all letters relative to this department to Eugene Randolph Smith, Headmaster, The Park School, Baltimore, Md. In answering an advertisement from a school or college, enclose your letter in an unaddressed stamped envelop, and enclose this with your letter to the editor. The blank envelop will be addressed to the school or college by the editor. Letters not enclosing postage cannot be forwarded or answered.

The Last Greek Geometer.

Thales, a philosophizing Greek merchant of the sixth century B. C., is said to have meditated on the very practical Egyptian geometry and to have begun to question nature and attempt to arrive, through philosophy, at the solution of practical problems. These meditations were continued during the legendary age of Pythagorus, growing more philosophic and cryptic, and where revealed expressed in a geometric dialect.

In Plato we have the union of philosophy with mathematics. Later Euclid, like Shakespeare, collects, edits and does some original work. However, the Greek philosopher believed experimental science beneath him, and the men in the street felt it to be blasphemous until Archimedes dared create the "Geometry of Measure." Appolonius followed immediately with the "Geometry of Order" and about 200 B. C. we have the "Golden Age of Greek Mathematics."

No disciple is able to fully appreciate and interpret the writings of either genius for five hundred years, when Pappus, collector, commentator, and investigator, kindles again the ancient fire of the Oracle of Delphi, exhausts the subject of conic sections, and indicates other lines of investigation.

Then all is dark for a thousand years, till finally Kepler adding to his *unusual assortment of knowledge* finds in a musty old book the completed investigation of an abstract problem studied lovingly by mathematicians, for more than eighteen hundred years, merely to satisfy a craving for knowledge. Being *widely* educated, he recognizes the key to one of the most important laws of nature.

Some attempt was made at about this time to discover or reconstruct the ancient and neglected Greek mathematics, but modern philosophers, thinking they followed Leibnitz and Newton, forgot that these masters had made a profound study of the methods of the ancient Greeks.

Novel ideas are apt to spring from comparisons of distinct lines of thought. It has seemed to the writer that distinctly fruitful results may come from the discussion and solution of problems that have been epoch-making.

On another page is an article on "The Five Platonic Bodies." The key is the porportion mentioned as proved in Pappus, of whose works Mr. Weaver is completing a translation and commentary. Can any of our readers obtain this proportion? We expect to publish from time to time other famous problems and invite further solutions, and we will also gladly receive articles suggesting other epoch-making problems, suitable for our general readers.

THE FIVE PLATONIC BODIES.

By James H. Weaver.

Pappus, Book III., Propositions 43 to 58, discusses the inscription of the five regular polyhedrons in a sphere.

Taking up the twenty-sided solid, we note that the twelve vertices are in four parallel circles, equal in pairs, the smaller circles circumscribing faces of the icosahedron. The larger circles also contain three vertices of the icosahedron, thus circumscribing larger equilateral triangles.

Pappus proves that:

Diameter of sphere: side of larger triangle: side of smaller triangle.

: : Side of pentagon: side hexagon: side decagon,

these three regular polygons being inscribed in any given circle.

It is now a very easy matter to obtain these equilateral triangles, and construct their circumscribing circles parallel on the sphere, and then to step off on them the vertices of the icosahedron.

Pappus further proves that these same parallel circles contain by fives the twenty vertices of the dodecahedron, the pentagonal face of dodecahedron being inscribed in the self same circle in which the triangular face of the icosahedron was inscribed, a larger pentagon keeps company also with the larger triangle before mentioned, the side of this larger pentagon being the diagonal of the smaller pentagon, and as the square of this line is equal to one third of the square of the diameter of the sphere, it is also the edge of the inscribed cube.

Now a plane passed perpendicular to the diagonal of the smaller pentagon will cut out a circle circumscribing the face of the cube and likewise the face of the octahedron. Moreover, the diameter of this last circle is the edge of the tetrahedron. It is evident that the dodecahedron, the cube, and the tetrahedron can be inscribed in the same sphere with the vertices coinciding. The icosahedron and the octahedron can be obtained

from the dodecahedron and the cube respectively by taking the mid-points of the faces as vertices.

Thus two small circles of the sphere perpendicular to each other on the edge of the pentagon inscribed in the smaller give as sides of regular inscribed polygons the edges of the five regular polyhedrons. The golden section or mean and extreme ratio is particularly prominent in the construction. Note that the connecting link is the edge of the cube which the ancient Greek philosophers called Earth,* the octahedron being water, the tetrahedron fire, the icosahedron air, and the dodecahedron the sphere of the universe. We will note this last contains the key to all the others.

It is to be noticed that not only the mid-points of the twelve faces of the dodecahedron are the twelve vertices of the icosahedron which can be whittled from it, but vice versa there is a

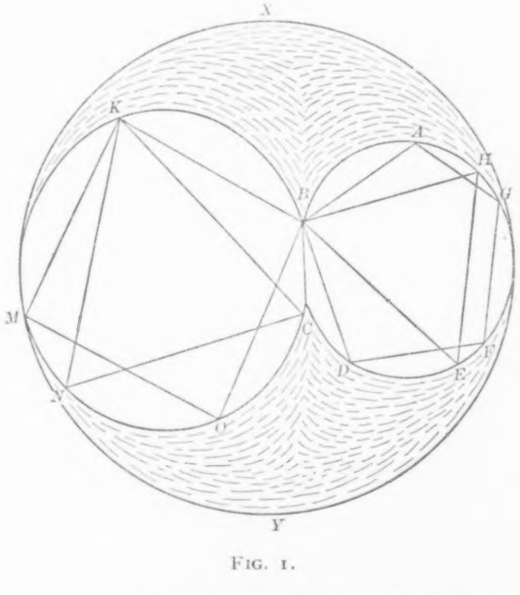

FIG. 1.

reciprocal relation. Now if we take the mid-point of any edge and successively the mid-point of the edge starting from the opposite angle, but not in the same face, we will form from either figure the regular octahedron, the mid-points of whose faces

* "Timæus" of Plato. Translation by Archer Hind, pages 190 seq.

give the cube (there is also a reciprocal relation here), and properly chosen vertices of the cube give the tetrahedron which is invariant.

In addition to these regular solids, Archimedes investigated several others that are equiangular and equilateral. He set up 13 such. They are as follows:

(1) One contained by 8 bases, 4 triangles and 4 hexagons.

Next there are three polyhedra of 14 bases.

(2) The first one of these is contained by 8 triangles and 6 squares.

(3) The second is contained by 8 squares and 8 hexagons.

(4) The third by 8 triangles and 6 octagons.

Then there are two of 26 bases.

(5) The first is contained by 8 triangles and 18 squares.

(6) The second by 12 squares, 8 hexagons and 8 octagons.

Then there are three of 32 bases.

(7) The first is contained by 20 triangles and 12 pentagons.

(8) The second by 12 pentagons and 20 hexagons.

(9) The third by 20 triangles and 12 decagons.

(10) Then there is one of 38 bases contained by 32 triangles and 6 squares.

Then there are two of 62 bases of which

(11) The first is contained by 20 triangles, 30 squares and 12 pentagons.

(12) The second is contained by 30 squares, 20 hexagons and 12 decagons.

(13) The last is of 92 bases, 80 triangles, and 12 decagons.

Now it is interesting to note that all these 13 may be cut from the five regular ones. For example, the first one may be cut from the pyramid by trisecting the edges and cutting off the corners. (2) may be obtained by taking the octahedron, bisecting the edges and then cutting off the six corners. The others may be obtained in the same way from the other solids, by cutting off corners or edges.

This seems to be conclusive evidence that the regular solids were whittled out of the sphere. At least such a thing was possible and a little study of the properties of the figures would lead to such a construction. On the other hand it is not possible to build up the regular solids by combinations of any of the five.

HIGH SCHOOL, WEST CHESTER, PA.

A STUDY OF THE RELIABILITY OF TEST QUESTIONS.*

By George Gailey Chambers.

This paper is a study of a test on deductive reasoning and a comparison of the results of that test with the teachers' marks in plane geometry. This test was given to 49 high-school girls who had just completed a half year's work in plane geometry covering during that time the first two books of Robbins & Somerville's text-book. Previous to their study of geometry they had studied algebra through simultaneous quadratics, spending on that 5 periods a week for one school year and 2 periods a week of $\frac{1}{4}$ of a school year. That was followed by 2 periods a week for $\frac{1}{4}$ of a school year in constructive geometry. In the case of 5 of these girls we have been unable to obtain the teacher's marks, so that in this paper the answers of only 44 girls are taken into account.

The Association of Teachers of Mathematics of the Middle States and Maryland has a committee at work investigating the results of geometry teaching. A preliminary report of this committee was presented to the Association about a year ago and was published in the recent December issue of the Mathematics Teacher. In this preliminary report it was stated that one of the first tasks that the committee had undertaken was that of preparing suitable non-geometrical tests to determine the results of geometry teaching as to reasoning ability. The writer of this paper happens to be chairman of that committee and the test discussed in this paper is the first one of a series of tests that is being given by the committee as suggested in that report. The purpose in this particular preliminary work is to determine the reliability of the questions used, rather than to test the results of geometry teaching. I am giving this discussion to this body now to inform them as to the progress of the work of the com-

* Read before the joint meeting of the New England Association of Mathematics Teachers and the Association of Teachers of Mathematics of the Middle States and Maryland, February 28, 1914.

mittee and especially to enable the committee to obtain frank criticisms of its work as it progresses. I should say, however, that it is not a report of the committee, and that any opinions expressed in this paper are opinions of the writer only.

One of the most pertinent problems before the educational world today is that of measuring results of educational processes. This general problem has been studied by a number of investigators and it has led to the introduction into educational literature of several technical terms such as, tables of distribution, frequency curves and coefficients of correlation. There is much literature on the subject but very little of it in those periodicals that come into the hands of mathematics teachers in general. It seemed wise therefore to present here a concrete example illustrating the meaning of some of the technical terms now current.

This is the set of questions used:

I. Do you discover any defects in the following reasoning, and if so, explain why it is defective?

The sidewalk was wet this morning. Therefore it must have rained last night.

II. If all the inhabitants of the Rahib Islands have blue tattoo marks on their bodies, then which of the following statements would necessarily be true, which could not be true, and which might possibly be true?

(1) All people who have blue tattoo marks on their bodies are inhabitants of the Rahib Islands.

(2) Some inhabitants of the Rahib Islands do not have blue tattoo marks on their bodies.

(3) No people with blue tattoo marks on their bodies live anywhere except on the Rahib Islands.

(4) Some of the inhabitants of the Rahib Islands have blue tattoo marks on their bodies.

III. A certain club wishes to select the evening for its regular weekly meeting which would be most satisfactory to its members. Accordingly the secretary wrote to each member, asking what evening would be most satisfactory.

Can you suggest another question which would have been better for the secretary to have asked?

IV. If a photographic plate be exposed to X-rays and then developed, black marks will be found upon it.

If upon developing a photographic plate you should find black marks upon it what would you conclude?

Also if you should not find black marks upon it what would you conclude?

V. If John agrees to join the football team provided Charles joins it, but Charles decides not to join it, what follows about John? If John joins, but Charles does not join, is John breaking his agreement?

A multigraphed copy of these questions was put into the hands of each pupil. No instructions were given except the following which was placed at the head of the multigraphed copy:

"Please write answers to the following questions, first reading them all before you begin writing. Answer them in any order you wish, numbering your answers to correspond to the numbers of the questions."

This test was given under the charge of two English teachers in the school and particular care was taken to see that there was nothing said or done to indicate in the minds of the pupils that there was any connection between this test and their work in mathematics. No inquiries in regard to the meaning of the questions were answered.

The answers to these questions were read carefully to discover the specific acts of correct deductive reasoning which appeared in the answers. The largest number of points made by any one pupil was 8, and the smallest number was 1, a point being scored for each act of correct deductive reasoning.

I will now give a resumé of the most common answers to the various questions, and also some significant but less common answers:

Question I.: Thirty-one stated that the sidewalk might have been wet from other causes and that therefore the reasoning was defective. A point was scored for each of these.

Three dealt with criticisms of the use of the pronoun "it" in the statement of the problem, declaring that it referred either to the sidewalk or to the morning, and that it was in that respect that the statements were defective.

Three stated that the two statements were defective because they gave the effect before the cause.

Two specifically declared that the reasoning was correct as given.

Question II. (1): Sixteen stated that it might possibly be true, or it is not necessarily true. A point was scored for each of these.

Six that it is not true because there are many people not inhabitants of the Rahib Islands who have blue tattoo marks on their bodies. From the evident premise in the mind of the pupil the reasoning is correct and a point was scored for each. These answers, however, show that this question is defective in that it is so easy for the pupil to add another premise.

Four that it is not true because some people not living on the Rahib Islands might have blue tattoo marks. This answer in itself is illogical.

Ten that it was necessarily true.

Eight that it could not be true.

Question II. (2): Thirty-one answered that this statement could not be true. A point was scored for each of these.

Three that it might possibly be true because foreigners visiting these islands would not have blue tattoo marks. Evidently the given statement was interpreted loosely in these cases; that is, that there were probably exceptions to it. With that interpretation the reasoning was logical and these questions were counted as correct. The defect was in the looseness of the interpretation and not in the reasoning.

Three that it might possibly be true because there may be a few who do not have blue tattoo marks, by being accidentally missed or by not believing in the "severe custom." In these cases also the pupils recognized that the true facts were contradictory if the two statements were taken as holding without exception. Evidently the reasoning from the premises in the minds of the pupils was correct.

Five that it might possibly be true without any indication as to any modification of the given premise. These were not counted as correct.

One that it would necessarily be true.

Question II. (3): Twenty-four replied that this statement might possibly be true. A point was scored for each of these.

Two that it is true.

Eight that it could not be true, but giving an explanation showing that they had added an additional premise to the given

one. Their reasoning was evidently correct from the premises in mind, and a point was scored for each.

Six that it could not be true, but giving no indication of any added premise.

Question II. (4): Nineteen replied that it is necessarily true. A point was scored for each of these.

Two that it might possibly be true.

Nine that it is not true because it implies that some of the inhabitants do not have blue tattoo marks. With this interpretation of the statement the logic is correct and a point was scored for each.

Ten that it is not true but giving no indication of their interpretation of the given statement.

Question III.: Five suggested that he might better have asked what evenings were not satisfactory. A point was scored for each.

One that he might have asked what evenings were open. A point was scored for this.

Twenty-one that he should have named one or possibly two evenings and asked the members whether they suited. These were not counted as correct.

One that the secretary should have asked the members to come out at an appointed time and then decide the evening for the meeting. This was not counted.

Eight suggested other words instead of the words "most satisfactory," but made no suggestions affecting the reasoning involved. These were not counted.

Question IV. (1): Twenty-eight concluded that the plate had been exposed to X-rays. These were not counted as correct.

Five recognized that there might have been other causes for the black marks. A point was scored for each of these.

Question IV. (2): Thirty-two concluded that the plate had not been exposed to X-rays. A point was scored for each of these.

Five that it had not been exposed to X-rays nor to anything else that might have caused black marks. A point was scored for each of these.

Question V.: Six answered that John would not be breaking his agreement. A point was scored for each.

Twenty-eight that John would be breaking his agreement. These were not counted as correct.

One of these stated that it should not be considered as the breaking of an agreement because the matter was not of such importance that to change his mind could be called breaking his agreement.

Another stated: "If John joins the football team but Charles does not join, then John is breaking his agreement provided the agreement was written and signed by both parties; if not, John is not doing anything wrong because he merely agrees to that and is eligible to break his word."

As far as the logic is concerned these last two answers are not different from the other six; but they are interesting as indicating the moral point of view of the pupil.

This leads to the following summary giving the number of pupils who scored on the various questions considering each of the four parts of question II. as independent questions, and likewise the two parts of question IV.

TABLE I.

Question.	No. Scoring.
I	31
II (1)	22
II (2)	37
II (3)	32
II (4)	28
III	6
IV (1)	5
IV (2)	37
V	7

No two pupils scored on both question III. and question IV. (1). Three scored on both question IV. (1) and question V. One scored on both question III. and question V.

These results may also be arranged as in the first two columns of the following table. Those two columns of this table constitute what is called a table of distribution.

They may also be represented by a frequency curve, as in Fig. 1.

This is drawn on the assumption that the difference in the amount of the ability in question between a pupil who made, for example, two points and a pupil who made one point is equal to the difference in the amounts between two pupils scoring any other two consecutive numbers.

TABLE II.

Number of Points Scored by One Pupil.	Number of Pupils Making the Corresponding Score.	Rank.
8	1	1
7	3	3
6	11	10
5	11	21
4	5	29
3	10	36½
2	2	42½
1	1	44

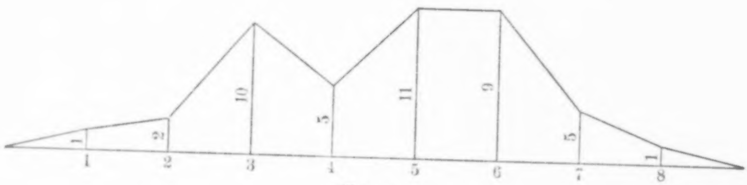

FIG. 1.

Table III. contains a similar table of distribution for the same 44 pupils based upon the teacher's mark in geometry.

TABLE III.

The Teacher's Marks.	Number of Pupils Receiving the Corresponding Mark.	Rank.
A	5	3
B +	5	8
B	11	16
C +	17	30
C	6	41½

A is the highest mark.

Fig. 2 is the corresponding frequency curve.

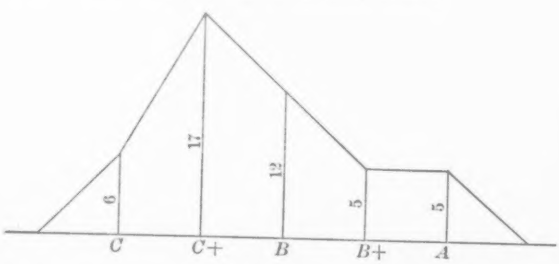

FIG. 2.

Theoretically a frequency curve should take the form shown in Fig. 3.

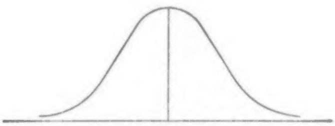

FIG. 3.

On comparing Fig. 1 with Fig. 3 we observe that the number of pupils scoring 4 points is abnormally small, but that otherwise the curve of Fig. 1 approximately coincides with a curve of the type shown in Fig. 3.

A similar comparison in the case of Fig. 2 shows that the number of pupils marked C + is abnormally large.

We will now study the correlation between the series of measures obtained from these test questions and the series of measures given as teacher's marks; that is, roughly speaking, the tendency that a pupil having a high measure in one series will also have a high measure in the other series; a low measure in one, a low measure in the other; and a medium measure in one, a medium measure in the other.

Since the two sets of measures are so essentially different, we must make use of ranks, that is, the numbers which give the positions of the pupils when they are arranged in a series according to merit. Referring to Table II., it is evident that the pupil making eight points should have the rank 1. It is also evident that the three pupils making 7 points each should have the ranks from 2 to 4, but we have no way of assigning these ranks to individual pupils. The best we can do is to assign to each the same rank. In this and the following cases the average of the real ranks has been assigned in Table II. In the same way the ranks have been assigned in Table III.

This leads to the following table giving the 44 pupils, with their corresponding ranks in the two series.

The mathematical theory of probability gives the formula

$$\rho = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)},$$

where d is the difference between the ranks of the same pupil in the two series, Σd^2 is the sum of the squares of those differences,

n is the number of pupils, and ρ is a measure of the correlation. To avoid fractions I have written the formula as follows:

$$\rho = 1 - \frac{3\Sigma(2d)^2}{2n(n^2 - 1)}.$$

The data of Table IV. gives $\rho = .48$.

TABLE IV.

Pupil.	Ranks from Teacher's Marks in Geometry.	Ranks from Results of This Test.	Differences in Ranks.	Squares of Twice the Differences.
E. B.	30	10	20	1,600
M. B.	30	36½	6½	169
R. B.	16	21	5	100
L. S. B.	8	3	5	100
M. E. B.	41½	21	20½	1,681
I. M. C.	30	29	1	4
M. J. C.	41½	29	12½	625
M. C.	30	36½	6½	169
H. A. D.	3	10	7	196
L. D.	16	10	6	144
M. D.	30	29	1	4
M. E.	30	36½	6½	169
M. F.	16	21	5	100
J. B. F.	30	21	9	324
J. F.	8	10	2	16
I. T. G.	16	10	6	144
F. H.	41½	29	12½	625
T. H.	41½	29	12½	625
S. J.	41½	36½	5	100
E. V. J.	16	10	6	144
M. M. L.	8	21	13	676
H. W. L.	16	21	5	100
R. G. L.	41½	44	2½	25
M. M.	16	3	13	676
E. M.	3	36½	33½	4,489
E. T. M.	8	10	2	16
E. M. M.	30	36½	6½	169
D. M.	30	21	9	324
R. N.	30	36½	6½	169
E. N.	3	10	7	196
E. O.	8	1	7	196
C. P.	30	21	9	324
A. P.	16	42½	26½	2,809
E. P.	16	36½	20½	1,681
O. R.	3	10	7	196
E. R.	30	10	20	1,600
G. R.	30	10	20	1,600
M. R.	3	21	18	1,296
H. M. S.	16	21	5	100
F. S.	16	36½	20½	1,681
D. V. S.	30	3	27	2,916
E. Y.	30	21	9	324
R. Z.	30	42½	12½	625
P. Z.	30	36½	6½	169

The use of ranks instead of actual measures has introduced an error. This can be at least partly corrected by the formula

$$r = 2 \sin\left(\frac{\pi}{6}\rho\right).$$

This gives $r = .50$, as the corrected coefficient of correlation.

A natural question to ask is as to the probability that the apparent correlation in this case is a real correlation due to a functional relation rather than to pure chance. This is answered by determining the probable error. Recourse again to the theory of probability shows that the probable error of r is given by the expression:

$$0.706 \frac{1 - r^2}{\sqrt{n}}.$$

In this case the probable error is .08. That means that it is an even chance that the true value of r is between $.50 - .08$ and $.50 + .08$, that is, between .42 and .58.

The chances are 16 to 1 against the true value of r differing from the above value, .50, by more than three times the probable error; that is, the chances are 16 to 1 that the true value of r is between .28 and .74.

Similarly the chances are 1,000 to 1 that the true value of r is between $.50 - 5 \times .08$ and $.50 + 5 \times .08$; that is, between .10 and .90.

It is true that the fact that there were so many groups of pupils with the same rank may increase the probable error, yet there is reason to think that notwithstanding that fact, the chances are large in favor of the true value of r being greater than .3. That means that we can feel reasonably certain that the traits measured by these two sets of measures are correlated; that is, as expressed before, there is a tendency for a pupil high in one series to be high in the other, and so forth.

As stated in the beginning, one purpose before me has been to give this body of teachers a concrete illustration of some of the applications of the theory of probability to experimental educational data. May I suggest that a very valuable course in any teacher's preparation is a course in the theory of probability with special application to educational statistics. Such a course is especially valuable to a teacher of mathematics.

My main purpose, however, was to determine whether this set of questions based on non-mathematical subject matter had any real value in measuring reasoning ability. The conclusion seems to be fully justified that it has a value for that purpose. It is also evident that some of the questions can be modified so that the set will serve as a still better means for that purpose. This has an important bearing on the general question of testing the results of geometry teaching, because such a test can be given to groups of individuals some of whom have not studied geometry, while others have studied that subject. The results can then be investigated to determine whether or not those who have studied geometry tend to rank higher than those who have not studied that subject.

Note.—Before reading this paper I gave opportunity for the persons present at the meeting to answer the same set of questions. Forty-eight persons answered them. I have marked these answers in the same way as I marked the answers of the pupils, with the following results:

Question I.: Forty-five stated that the sidewalk might have been wet from other causes and that therefore the reasoning was defective.

Question II. (1): Forty-five stated that it might possibly be true or it is not necessarily true, one that it was necessarily true, two that it could not be true.

Question II. (2): Forty-eight answered that this statement could not be true.

Question II. (3): Forty-six answered that this statement might possibly be true, two that it could not be true.

Question II. (4): Forty-four replied that it is necessarily true, one that it might possibly be true, two that it is not true.

Question III.: Twenty-five suggested that he might better have asked what evenings were not satisfactory. Ten that he might have asked what evenings were satisfactory. One that he should have named one or possibly two evenings and asked the members whether they suited.

Question IV. (1): Six concluded that the plate had been exposed to X-rays. Thirty-six recognized that there might have been other causes for the black marks.

Question IV. (2): Thirty-four concluded that the plate had not been exposed to X-rays.

Question V.: Twenty-eight answered that John would not be breaking his agreement. Seven that John would be breaking his agreement.

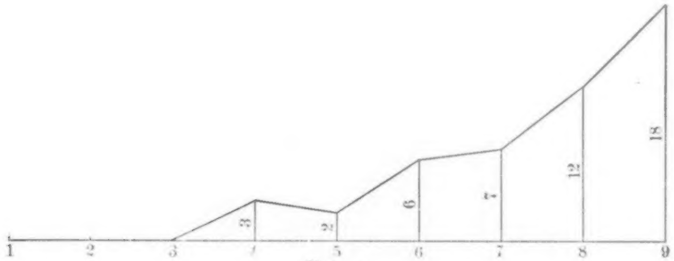

FIG. 4.

The following table gives a summary of the number of persons who scored on the various questions. This corresponds to Table I. of the above paper.

TABLE V.

Question.	No. Scoring.
I	45
II (1)	47
II (2)	48
II (3)	46
II (4)	46
III	35
IV (1)	36
IV (2)	34
V	28

These results also lead to the following distribution table which corresponds to Table II. above.

TABLE VI.

No. of Points Scored by One Person.	No. of Persons Making the Corresponding Score.
9	18
8	12
7	7
6	6
5	2
4	3

The corresponding frequency curve is shown in Fig. 4, which corresponds to Fig. 1 above.

BUSINESS ARITHMETIC VERSUS ALGEBRA IN THE HIGH SCHOOL.

By George H. Van Tuyl.

In bringing this discussion before you to-day, it seems wise to state the reason for its selection. This explanation is offered, not in any sense as an apology for the subject or its discussion, but merely to state my position, lest I should be misunderstood.

When your President first wrote me he asked if I would speak on some phase of business arithmetic. I hardly knew what phase of business arithmetic would be interesting to a company of mathematics teachers. After some reflection it occurred to me that I had read an address delivered by Dr. David Eugene Smith, in which he stated as the first of a list of problems confronting mathematics teachers, this problem, which, to put it in his own words, reads: "The first of the great questions that confront us at the present time relates to the very existence of secondary mathematics in our curriculum." Dr. Smith admits he has no solution. I am not here claiming to solve problems which the gentleman quoted cannot solve. Nor is it in my thought to attack mathematics or mathematics teachers. We all recognize the value of mathematics in certain lines of work. The thoughts I may have to present are for the purpose of adding to the discussion. If from my remarks anything that will have a tendency to help solve the question of what are the most valuable subjects in a high school curriculum, can be gained, I shall be content.

In the first place let us keep clearly in mind the fact that our topic and its discussion have to do only with secondary or high school work. In the second place we must remember that only about five per cent. of our high school students ever go to college. Our discussion, then, must be carried on with a view to meeting the needs of those whose school days end at, or before, graduation from a secondary school. I claim we have no right in these days of the free public school to frame a course of study to meet the requirements of one student while we neglect the immediate and pressing need of nineteen others.

It is only proper, then, that at this point, we should formulate some notion of the purpose of the high school curriculum. For far too many years the main objective has been to prepare for college entrance. It is only within comparatively recent years that the demand to prepare for life's work has been heard and heeded. One by one subjects whose presence in the curriculum could not be justified even by their most ardent supporters, have been dropping out, and others of greater practical value have taken their places. The trend of events points clearly the way. The public is demanding that for the large investment in high school education there shall be a reasonably adequate return in the form of practical, usable knowledge in the possession of the boys and girls who are graduated from our high schools.

With these thoughts in mind, let me submit a question, trite though it may be, viz., "What is the educational value of secondary mathematics, or more particularly, of algebra, for students who do not go to college?" Prof. Young, in his "The Teaching of Mathematics," tells us that the study of mathematics exemplifies most typically, clearly and simply certain modes of thought," one of which modes, he says, "is the ability to grasp a situation, to seize the facts, and to perceive correctly the state of affairs." Of algebra, in particular, he mentions four functions only three of which will I call to your attention. (1) "To establish more carefully and to extend the theoretic processes of arithmetic. (2) To strengthen the pupil's power in computation by much practice as well as by the development of devices useful in computation. (3) To develop the equation and to apply it in the solution of problems of a wide range of interest, including large classes of problems often treated in arithmetic, as well as problems relative to geometry, to physics, and to other natural sciences."

It is unnecessary for me to attempt to tell you what are the purposes or the value of your own subject. The quotations just cited are for the purpose of establishing some basis of comparison between business arithmetic and algebra.

The first question that presents itself to my mind is, "Does mathematics in general, or algebra in particular, give one the 'ability to grasp a situation, seize the facts, and perceive correctly the state of affairs' better than does the study of business arith-

metic?" I am aware of the fact that an isolated case here and there proves but little, yet I would like to mention one or two conspicuous illustrations of the failure of mathematics to enable students to "grasp the situation." It was my privilege recently to give instruction in business arithmetic to two young ladies, one of whom had passed differential calculus, while the other was planning to specialize in mathematics for her Ph.D. degree. One of them brought me this problem: "A man bought two horses at the same price; he sold one of them at a loss of 10 per cent., but for the other one he received a sum sufficient to make up his loss on the first horse and 20 per cent. of that loss in addition. He thereby gained \$5.00. What was the cost of each horse? What was the loss on one, the gain on the other?" I do not recall the solution she presented with it. It was an attempt at an algebraic solution with the usual X for the unknown value. Aside from the original statement of "X = the value of each horse," she had not a single correct equation in her solution, and needless to say, she did not arrive at a correct conclusion.

Let us apply a little business arithmetic to the problem in question. Note the conditions. Two horses cost the same amount; one was sold at a loss of 10 per cent., and the other at a sufficient gain to make up the loss on the first and 20 per cent. of that loss additional. The gain on both was \$5.00. The problem reduces to this simple equation: 20 per cent. of 10 per cent. of the cost of 1 horse = \$5.00. Solving we have 20 per cent. of 10 per cent. = 2 per cent.; that is 2 per cent. of the cost of one horse = \$5.00. Since 2 per cent. of the cost is \$5.00, the entire cost is 50 times \$5.00, or \$250. The remainder of the solution does not need discussion here. The problem as you perceive is a very simple one, and is solved mentally with ease. Surely mathematics through calculus did not give "ability to grasp a situation, to seize the facts, and to perceive correctly the state of affairs."

The other young lady was even less able to solve problems in business arithmetic, though she could talk glibly of quadratics, surds, negative exponents, irrational quantities, etc.

I have at the present time a class of young men who have had from two and one half to three years of high school mathematics, who were unable to solve or even to state, when they entered my

class, such a simple problem as this: "A house and lot cost \$5,000. If taxes, repairs and other expenses amount to \$180 per annum, what rent per month must the owner receive to clear 6 per cent. on his investment?" Had this been a condition afflicting only a small portion of the class, it might have been overlooked as the usual condition of affairs, but fully 75 per cent. of the class were unable to solve the problem quoted and others of like difficulty. This is a condition not at all peculiar to the present class, but it is one which confronts me from term to term. In the face of these conditions one begins to wonder what has become of the "ability to grasp a situation" which a study of mathematics is said to give.

Some years ago I taught business arithmetic in a school in which but very few of the students had ever studied any other branch of mathematics than arithmetic in the elementary school. These boys and girls did better work in arithmetic than do the boys I now have. Do not understand me to say that they did better work becat e they had no knowledge of mathematics; what I mean is that those who *have* had the mathematics do not show by their ability to solve problems that mathematics has helped them.

Recently I had the pleasure of visiting a school in a neighboring city. Naturally I visited an arithmetic class. The students were finishing their second year in high school. They had studied algebra the preceding year and a half. The arithmetic was being taught by the use of algebraic formulæ, logarithms, etc. I left the class with the firm conviction that if the year and a half spent in the study of algebra could have been devoted to business arithmetic these students would have been far better equipped to interpret and solve problems in the business world than they are with the knowledge of algebra they displayed.

What is the matter? Why is it that so uniformly the speaker has found students not possessing the ability which their study of mathematics is supposed to give them? Is it poor teaching, or is it possible that the study of mathematics does not give what is claimed for it? Let me venture an opinion. It looks this way to me: The student of algebra spends a year or more studying unknown quantities, negative quantities, surds, and to him a lot of things that are *ab*surd, and at the end of that time he has

mixed up in his mind a lot of unknown "unknowns." How many high school students can give an intelligent exposition of imaginary quantities or complex roots? These and many other topics common in algebra are entirely outside the experience of the pupil. There is little or nothing in his mind to which he can relate or attach the new matter. He has no means of classifying the new information he is receiving. The result is a jumble, and consequently, when the student wishes to apply his knowledge to the solution of practical problems he is at a loss to know how to proceed. In other words, the time required to train a student to solve problems intelligently by the algebraic process is greater than is allowed for such training in the high school, or is warranted by the benefits derived therefrom.

A second reason, in my opinion, is that in the study of algebra, too much attention is given to theorizing on possible solutions for impractical conditions, and to solving equations that are already prepared for the student, or, to put it differently, too *little* attention is given to solving practical problems. In one of the recently published algebras containing 447 pages there are less than 500 problems for solution. By problems I mean exercises requiring the student to formulate his own equations from the data given. The great majority of the problems are of no value save as exercises in calculation. They are made for no other purpose than to fit the theories that have been discussed in the earlier parts of the chapters. A student may be skillful in solving problems in simultaneous equations or in quadratics, but such skill does not aid him materially in solving business problems.

Professor Young says of algebra that its function is "to establish more carefully and to extend the theoretic processes of arithmetic." I have yet to find any process in arithmetic required in business that cannot be sufficiently and thoroughly established without resorting to algebraic processes. The demand of the day is not for theoretic processes, but for practical processes. The process that produces results the most quickly and the most easily is the one that wins. A study of negative quantities is not necessary to convince a boy that, if he earns $10.00 and spends $5.00 of it, he has $5.00 left; or if he starts from his home and walks 8 miles north and then 4 miles south, he is 4 miles from his

home. Neither does he need to study simultaneous equations to solve this problem: "A laborer engaged to work 48 days at $2.00 a day and his board. But for every day he was idle he was to pay $1.00 for his board. At the end of the time he received only $42.00. How many days did he work?" All such a problem needs is the application of a little common sense, and it can be solved mentally far more quickly and easily than by simultaneous equations.

The third function of algebra, according to Professor Young is "to develop the equation and to apply it in the solution of problems, etc." No one favors the use of the equation in solving problems more than does the speaker. I fear too much attention is given to developing the equation and too little to its application. A business arithmetic was recently published, in which the author, a former teacher of mathematics, tried to feature the algebraic equation as a mode of calculation. Four pages were used in developing the equation, and then he left it suspended in mid-air and did not apply it to a single problem.

Enough has been said concerning the algebraic side of our topic. Let us now give some thought to business arithmetic. The question is sometimes asked, "What is business arithmetic? How does it differ from arithmetic? I would answer the first question by saying, business arithmetic is the arithmetic of business. So far as the calculation side of the subject is concerned it has for its object the solving of problems and the making of calculations by up-to-date business methods, which is only another way of saying that solutions and calculations are made by the easiest, simplest, and shortest methods possible. Ordinary arithmetic is coming more and more to approach this standard. The difference between the two has been more in the kind of problems and in the method of calculation, than in anything else.

Business arithmetic has, then, as one of its objectives, skill in calculation. Too many people seem to think that skill in calculation is the main objective, that the business arithmetic teacher is training only for mechanical celerity, and that the business arithmetic course does not develop power "to grasp a situation, to seize the facts, and to perceive correctly the state of affairs." Far be it from me to hold up any such standard. In the first place it would be ridiculous, and in the next place its attainment

would be impossible. Skill in calculation (I shall use the term calculation not merely for the mechanical process of performing the fundamental operations, but for the entire process of interpreting and solving problems) is not a cause of power, but the result of power. Skill in calculation depends upon ability to interpret a problem, power or ability to apply the principles of business and of arithmetic to the problem, and ability to see the relation of numbers. Facility in calculation is the outward evidence of inward power. A second objective, then, of teaching business arithmetic, is the development of power. How shall we develop the power that leads to skillful calculation? You know as well as I do that pupils entering high school are not able to calculate with ease and accuracy. I have tried to tell you that when you turn them over to me for the study of business arithmetic they cannot interpret problems. Can we make ready calculators of our boys and girls by the study of business arithmetic? I believe we can. What are the requisites? First, time. Sufficient time must be given in which to teach the subject. The idea that some people have that in eight years of elementary school a boy has learned or ought to have learned all the arithmetic he needs, is indicative to my mind of only one thing, and that is that the person holding such ideas is entirely ignorant of the content of arithmetic. In many high schools one half a year is given to the subject. That time is entirely inadequate. Daily recitations for from one to two years are needed in the first half of the course. Later daily recitations for at least one half a year are required to take up topics too advanced for earlier discussion. A second requisite is qualified teachers. Here again, an erroneous impression seems to prevail. Owing to an incorrect notion concerning business arithmetic, many seem to think that anybody can teach the subject. In some schools business arithmetic is taught by teachers of modern language, or of English, or of any other department having a teacher to spare, while in other schools, the subject has been turned over to the mathematics department. With all due respect to the teachers of mathematics, I believe such a method of procedure to be unwise, and that for one reason only—mathematics teachers are not teachers of business arithmetic. Is it any wonder we get poor results?

I have not the authority to grant enough time for the study of business arithmetic, nor can I provide a sufficient number of qualified teachers to teach the subject. So let us consider a third requisite for the development of power. It is method. It is not in my thought to discuss methods as laid down in the pedagogical books, but to outline if I can by specific instances how we may proceed to develop power. I have yet to find the class that did not need drill in the fundamentals. Classes as a whole are slow and inaccurate; they are slow because they are using "long hand" methods of calculating when they ought to be using "short hand" methods. They are inaccurate partly because they are using cumbersome methods, and partly because they do not realize the importance of accuracy.

The first thing to do is to arouse interest in and enthusiasm for the subject. One way, and I believe the best way, of doing this is to start the class on such a topic as aliquot parts. Use simple exercises at first, explaining reasons and underlying principles. As the exercises increase in difficulty it will be observed that the subject divides into two parts, viz., direct aliquots or aliquants, as $.33\frac{1}{3}$ and $.37\frac{1}{2}$, and the indirect aliquots and aliquants, as $.11\frac{1}{4}$, $.13\frac{1}{3}$ and $.13\frac{3}{4}$. It is just as easy to multiply by $.13\frac{1}{3}$ as by $.12\frac{1}{2}$. As soon as a pupil begins to see the opportunities there are for abbreviated forms of calculation, and begins to study the relation of numbers for himself, he is on the high road to power that gives skill in calculation. There are a large number of direct and indirect aliquots and aliquants, but to use them intelligently requires a careful study of the relation of numbers. Let me give just a few instances to illustrate my meaning. Probably the most common numbers of which the aliquot parts are used are 100 and 60—100 as 100 cents in a dollar, or as 100 per cent. in percentage, and 60 in interest calculations. Let us consider some of the parts of 100 cents or $1.00. Let it be required to find the value of 48 articles at $11\frac{1}{4}$c. each. $11\frac{1}{4}$c. is equal to 10c. plus $\frac{1}{8}$ of 10c. Taking advantage of the commutative law of multiplication all we need to do is to point off 1 place in the 48 and add $\frac{1}{8}$ of the result, which gives a value of $5.40. If the problem were to find the value of 48 articles at $13\frac{3}{4}$c. each, we would first think of $13\frac{3}{4}$c. as $12\frac{1}{2}$c. plus $\frac{1}{10}$ of $12\frac{1}{2}$c. The result, then, is found by taking

$\frac{1}{8}$ of 48, which is 6, and adding $\frac{1}{10}$ of the 6 to itself, which gives 6.60, that is $6.60. Again let it be required to find the value of 48 articles at 13½c. 13½c. equals 12½c. plus 1c. The result is equal to $\frac{1}{8}$ of 48, which is 6 (that is $6.00) plus 48c., hence $6.48. You will observe that the prime object in such exercises as these is not the mere finding of the value of a given number of articles at a given price, but a study in the relation of numbers, which as I have already stated is one of the requisites of skill in calculation.

Similarly, there is great opportunity for a study of the relation of numbers in the use of the aliquots and multiples of 60. Let me give an illustration or two. For the purposes of interest calculations 60 days are regarded as equal to two months. Two months is $\frac{1}{6}$ of a year. Hence at 6 per cent. interest, the interest on $1.00 for 2 months (or 60 days) is $.01. Since the interest on $1.00 for 60 days at 6 per cent. is $.01, it is $.01 on each of any number of dollars for the same time and rate. Therefore to find the interest on any sum of money for 60 days at 6 per cent. point off 2 places, or remove the decimal point 2 places to the left. Having the interest for 60 days, it can be readily found for any number of days. For 30 days it would be half of the interest for 60 days; for 15 days, ¼; for 20 days, ⅓; for 75 days, add ¼ of the interest at 60 days to itself; for 16 days, take $\frac{1}{6}$ for 10 days, and then $\frac{1}{10}$ for 6 days, and add the results for 16 days, etc. By the commutative law of multiplication the interest on $600 for 79 days at 6 per cent. is found by pointing off 1 place in the days, hence $7.90. If time permitted I could give almost endless illustrations of the use of this principle.

Now let us consider a topic in fractions as a study in the relation of numbers. Suppose it is desired to multiply 16¼ by 8½. Probably 8 students out of 10 who enter high school will, if given this operation to perform, reduce each mixed number to an improper fraction. They will then multiply 65 by 17, and divide the product by 8, using in most cases the long division process. What a waste of time, of effort, of opportunity! How much better is the four step process! Look at the numbers. Think the product of the fractions, of the cross products, and finally the products of the integers, adding the parts as you proceed. The result is 138⅛. Once more, let us multiply 7⅓ by 5 ⅓. Think

the product of the fractions. The cross products ($\frac{1}{3}$ of 5) + ($\frac{1}{3}$ of 7) = $\frac{1}{3}$ of 12; the product of the integers is 35, hence $39\frac{1}{9}$, as the completed product. This is the kind of drill that will develop power that will be usable in the solution of problems. But students must be directed in this work; they will not discover these methods for themselves.

The topics illustrated, together with many others that might be mentioned, have to do with the fundamental operations. Proper kind of drill in these subjects arouses interest in and creates a liking for arithmetic. Calculations of this kind require thought, and keep the pupil ever on the alert for new, short methods. Every operation becomes a problem in method of calculation. Soon the pupil becomes an independent investigator in the realm of practical methods of calculation. This is the kind of work from which mental discipline comes.

Let us now come to the question of problems. We observed that, in algebra, problems were made to fit certain theories of calculation regardless of the content of the problem. It is "putting the cart before the horse." In business arithmetic, the chief emphasis is laid on providing practical problems, and then devising practical methods of solving them. To solve problems successfully requires a knowledge of, or the ability to do, four things. First, one must be able to read plain, simple English understandingly. Many a failure in arithmetic, and in mathematics, has its roots right here. If one cannot from the printed page ascertain what data are given, and what result is required, his case is hopeless until such defect is remedied. Second, one must be able to perform the fundamental operations with ease and accuracy. We have already spoken of this phase of the work, and it is assumed that, by the time problems are presented for solution, students will be somewhat proficient in the mechanical processes required. The third and fourth things necessary for successful solution of problems, I shall discuss together, as I consider them indissolubly connected. They are the fundamental principles of business and the fundamental principles of arithmetic. As the principles of business let me mention two or three merely as illustrations: Cost is the basis on which gain or loss is reckoned; commission is computed on the prime cost or the gross sales; bank discount is reckoned on the maturity

value of a note for the exact number of days (with very few exceptions) from date of discount to date of maturity. A knowledge of these principles and many others that might be cited is absolutely necessary to interpret problems. As an illustration of the principles of arithmetic, I will mention but one. The product of two factors divided by either of them will give the other one. This one principle underlies every business problem involving a division. The application of this principle results in the use of the equation, the development and application of which Prof. Young says is one of the functions of algebra. Personally I see no need for algebra in this connection.

Let me illustrate the use of the principles and of the equation in a few simple problems. Of a consignment of eggs 5 per cent. were broken. For the remainder \$39.90 was received at 35c. per dozen. How many dozen eggs were in the consignment? If 5 per cent. of the eggs were broken, 95 per cent. of them remained to be sold. The problem reduces to two equations:

1. ? × \$.35 = 39.90.

Dividing the product, \$39.90 by the factor, \$.35, gives 114, the number of dozen sold.

2. 95 per cent. of ? doz. = 114 doz.

Dividing 114 doz. by .95 gives 120 doz., the desired result.

Take a second problem: A jobber buys hats at \$48 less 20 per cent. and 16⅔ per cent. per dozen from the manufacturer. At what price per dozen should he mark them to gain 35 per cent. after allowing 16⅔ per cent. and 10 per cent. to his customers?

1. The series 20 per cent. and 16⅔ per cent. = a single discount of 33⅓ per cent.
2. 66⅔ per cent. of \$48 = \$32, net cost per dozen.
3. 35 per cent. of \$32 = \$11.20, gain.
4. \$32 + \$11.20 = \$43.20, net selling price.
5. The series 16⅔ per cent. and 10 per cent. = a single discount of 25 per cent.

Hence the net selling price equals 75 per cent. of the marked price.

6. 75 per cent. of M. P. = \$43.20.
7. \$43.20 ÷ .75 = \$57.60, the marked price.

All through this problem are interwoven the principles of business and the principles of arithmetic. The solution is given in full to illustrate the use of the equation. Personally, I prefer a much briefer solution. It is this:

3) \$48
16, discount.
32, net cost.
11.20, gain (35 per cent. of net cost).
3) 43.20, net selling price.
14.40, discount to purchasers.
\$57.60, marked price.

Every principle and every equation used in the other solution is here used but in a shorter, more practical manner. When the student can solve problems by this shortened process, it is an evidence of clear thinking and careful application—an evidence of inward power.

A man desires to buy 150 shares of stock paying 6 per cent. dividends at a price to yield 5 per cent. on his investment. At what price should he buy the stock, and how much will the 150 shares cost him? (Make no allowance for brokerage.)

1. 6 per cent. of \$100 = \$6, dividend on 1 share.
2. 5 per cent. of cost of 1 share = \$6.
3. \$6 ÷ .05 = \$120, cost of 1 share.
4. 150 × \$120 = \$18,000, cost of 150 shares.

Here again we find the use of both the principles of business and of arithmetic.

If our students possess the ability to read intelligently, can add, subtract, multiply, and divide correctly, and have a knowledge of business principles and customs, and of arithmetical principles, they can solve any business problem presented to them. In two of these particulars, in my opinion, algebra fails. It gives no training in business principles, nor in the proper arithmetical principles. As to the equation, it can just as well be taught and used in arithmetic as in algebra.

In conclusion, then, let me say that, so far as my experience goes, I do not find that a study of mathematics, or of algebra in particular, gives ability to solve business problems. This is due chiefly, I believe, to the following: First, in algebra, symbols

are used for the thing or number for which the symbol stands. I see no advantage for business purposes in using an imaginary thing or number for the real, concrete thing itself, as represented by its correct name or value. In the second place the problems in algebra are artificial, impractical, and largely useless for any business purpose. And thirdly, one does not learn in algebra the fundamental principles of business or of arithmetic as applied to business, nor does he learn the abbreviated methods of calculation so important in the business world.

What shall we say then; shall algebra continue to be required in the high school that mathematics may abound, or shall we lay aside our prejudices, and investigate the merits of business arithmetic? Let me be a radical for just a moment, and suggest that business arithmetic be made a required subject throughout the first year of the high school. If necessary, eliminate part of the mathematics, and begin the study of algebra in the second year of the course. This would give those students who leave at or before the end of the first year something practical that they could use upon leaving school. It would also give a far better type of student, on the average, for the study of algebra. This fact, together with the fact that they would have a better understanding of arithmetic, which is the basis of algebra, would largely compensate for the loss of the first year in the study of mathematics. I do not expect you to agree with me in this, but I firmly believe it would result in decided advantage to our students.

The High School of Commerce,
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DISCUSSION OF MR. VAN TUYL'S PAPER.

By W. S. Schlauch.

Mr. President: I have listened with a great deal of interest to the paper just read, because for the past year and more I have been investigating the mathematical calculations involved in business. Visits to such institutions as the National City Bank, The Columbia Knickerbocker Trust Co., The Seamen's Bank for Savings, Spencer Trask and Company, fire and life insurance companies, exporting houses and department stores have revealed some interesting facts.

In the first place, let me say that if we are to train the boys and girls in the commercial high schools to occupy only clerical or recording positions, the only mathematics we should give them is a drill in the four fundamentals, interest, and discount. This would enable the student to keep a minor position, but would not enable him to seize an opportunity to move up into the class of employes who have to do with organization and control of business, as distinguished from mere recording or clerical work.

In one respect, I think the attack of Mr. Van Tuyl on elementary algebra of the conventional type is justified; namely, the subject matter found in the problems offered as applications of the algebraic theory. However, commercial algebra, specifically pointing out the applications of algebra to business, can be made more interesting to the student, and more effective as an engine for calculation, than what now passes under the name of business arithmetic.

To illustrate by instances that I found in my visits, certain short cuts in calculation employed by book-keepers and accountants are more easily mastered by students who have had the algebraic generalization than by those who have not. For example: A student who understands the algebraic theory of addition of fractions, has no trouble in seeing that

$$\frac{1}{a}+\frac{1}{b}=\frac{a+b}{ab}$$

leads to the short cut rule for adding fractions whose numerators are 1, as

$$\frac{1}{7}+\frac{1}{11}=\frac{7+11}{77}=\frac{18}{77}.$$

The algebra lays the emphasis on the *process* and shows by the use of letters which represent *any* numbers that the process can be *generally* applied.

Again, aliquot part multiplying of which Mr. Van Tuyl makes so much in his text-book, giving 12 pages to its treatment, can be unified for a student who has had algebra and condensed into the formula:

To multiply any number A by n/d of 10^k, add K zeros and take n/d of the result.

Thus, to multiply 328 by $37\frac{1}{2}$, or by $\frac{3}{8}$ of 10^2, add 2 zeros and take $\frac{3}{8}$ of the result.

In allowing interest on daily balances, at the end of the month, the book-keepers in the large banks take advantage of the algebraic principle of common factor addition:

	Balance × Rate × $\frac{1}{365}$.
$3ax$	$35000 \times .02 \times \frac{1}{365}$
$5ax$	$45000 \times .02 \times \frac{1}{365}$
$7ax$	$72000 \times .02 \times \frac{1}{365}$
$4ax$	$48000 \times .02 \times \frac{1}{365}$
Sum, $19ax$	....
	....
	Sum of Balances $\times .02 \times \frac{1}{365}$

The boy who has studied only arithmetic has trouble in seeing *why* it is not necessary to extend each item and take the sum of the separate days' interests.

Even the subject of interest is mastered more completely when generalized in algebra than in arithmetic. Each case in arithmetic is presented to the student as a *particular* and *separate* case. Algebra generalizes all into a formula and teaches the student to solve the formula to derive the converse cases.

Thus, (1) $i=prt$

and (2) $a=p+prt$

comprehend the whole subject of simple interest.

In fact, the men in the discount cage of the largest bank in America follow formula (1) in deriving a short rule for discounting notes when the discount rate is announced in the morning. For example, when I was in the bank one afternoon, it was 4½ per cent., and their rule was derived as follows:

$$r = \frac{9}{200}, \quad t = \frac{1}{360} \quad \therefore \quad i = p \times \frac{9}{200} \times \frac{1}{360} = p \times \frac{1}{8000}.$$

Therefore, *point off 3 places and divide by 8 for a day's interest.* In discounting a note for 24 days, they pointed off 3 places and multiplied by 3. A 30 day note for \$45,000 was discounted:

$$\begin{array}{r} 45 \\ 30 \\ \hline 8)\,1350 \\ \hline \$168.50 \end{array} \text{ Discount,}$$

although the clerk did not write as many figures as are used here to show the process.

The point I am making here is, that the generalization enables the discount clerk to derive his own "Bankers' Rule," and he does not find the interest or discount by the "Bankers' 60 day Rule" and then take off ¼, as the business arithmetics teach him he should. His method is algebraic and shorter.

Tables and efficiency devices I found to be in common use where repeated calculations are involved, and often the principle of calculation by which the table was made was algebraic.

The bond calculators used tables in determining the basis price of a bond, but they were of the opinion than in a High School of Commerce the method of calculating the basis price should be taught, and that a student should be taught *how to make such a table,* so that it would not be merely a mechanical process when he used it. This calculation involves only geometrical progression to derive the formula for the present value of an annuity, and logarithms to save labor in using the formula. Most business arithmetics publish tables of present values of annuities and bond tables, but the student uses them without understanding how they are derived.

For example, the basis price that can be paid for a bond bearing 5 per cent. interest, maturing in 14 years, to yield 4 per

cent. *can* be calculated by arithmetic when no table of answers is available, by using *14 long division processes.* It is calculated algebraically as follows:

The difference of rates will yield an annuity of $10, the present value of which can be used as a premium in purchasing the bond.

$$A_0 = \frac{s}{r}\left[1 - \frac{1}{(1+r)^n}\right]$$

gives the present value of an annuity *s*, at rate *r*, due in *n* years.

Substituting,

$$A_0 = \frac{10}{.04}\left[1 - \frac{1}{1.04^{14}}\right]$$

the premium that can be paid for the $1,000 bond bearing 5 per cent., to yield 4 per cent. interest.

Graphs are best studied algebraically. Many corporations now demand graphs, cartograms, pictograms and other illustrations of a quantitative kind. Algebra is needed more and more in business.

From algebraic theory, I have worked out an alinement chart by means of which a paymaster can calculate wages due when different rates for time and overtime are allowed, by simply connecting the hours time with the hours overtime by a line, and reading the total wages due on the axis of wages.

I might go on indefinitely, but the point I make is that all students in commercial high schools should have enough drill in the first year in arithmetic to bring them to the level of maximum efficiency in handling the fundamental operations, and then should take commercial algebra for the generalized control it gives over the number system, and because investments, interest, insurance, exchange and other commercial calculations can be best mastered from the algebraic side.

High School of Commerce,
New York City.

REPORT OF THE ARITHMETIC COMMITTEE.

INTRODUCTORY.

Perhaps no subject in the school curriculum, with a possible exception of English, has received as much attention as arithmetic. There is a wealth of text-books of wide variety on the topic, as well as an abundance of pedagogical and philosophical works. If we add to this, the multitude of articles published in the various journals, we are at once convinced of the diffculty of properly evaluating the results thus far attained and of making correct deductions from them.

During the more recent years, the most popular catch words in business and professional circles have been "efficiency" and "standardization." The latest profession is that of the efficiency engineer. If his activity is focused on a city system or upon a university, he is said to make a *survey*. Less pretentious are numerous investigations and experiments, many of which have involved great expenditures of time and thought, which are increasingly abundant in pedagogical journals.

Taking these facts into consideration, your committee has reached the conclusion that the time and funds at its command will not warrant the committee in inaugurating additional experimentation, nor is the best current opinion so accurately focused as to justify your committee in outlining model courses.

We believe we can render the wisest service to the Association at this time by calling attention to a few of the larger investigations that have recently been made; by reviewing contributions of conspicuous value that have lately appeared in the journals; and by making a few suggestions and recommendations growing out of the work of the committee.

* Report of the Arithmetic Committee adopted by The Association of Teachers of Mathematics in the Middle States and Maryland at the New York Meeting, November 28, 1914.

The Preliminary Report.

In an earlier report published in the *Mathematics Teacher*, Vol. V, No. 3, your committee gave abstracts of the replies of twenty-five leaders of educational thought to two questions:

I. What is the prime reason for teaching arithmetic in Grades V to VIII as our schools are generally organized?

II. What is the greatest need for reform if we are to accomplish this result?

While the answers to these questions were naturally worded somewhat differently, the thought contained was quite uniform:

I. "To acquire and retain speed and accuracy in the fundamental operations taught in the earlier grades and to show how these operations may be applied to problems of daily life, both of civic and local interest."

II. "The substitution of present day material, adjusted to actual business practice, for useless and antiquated problems,—simplification on practical lines."

In most schools, the work of the first four grades comprises the four fundamental operations. In some schools some knowledge of fractions is added to this ground. Courses of study in the later grades were formerly planned on the hypothesis that the elementary operations were completely mastered in the first four grades. Recent investigations in large city systems have shown that this is not the fact. There is a wide divergence in the ability to add, subtract, multiply, and divide in all the upper grades and in the high school. In an eighth grade for instance, may be found children, who as far as arithmetic is concerned, belong to all of the seven grades below.

For this reason, newer courses of study and recent text-books provide for review drill to cultivate speed and accuracy. There has also been great improvement in the content of material used. Problems dealing with modern business, industrial and social environment have largely given place to the worn out problems of the older text-books.

Experimental Studies.

A complete list of experimental studies is beyond the scope of this report. An excellent summary of such tests may be found in Dr. Ernest C. McDougle's "Contribution to Arithmetic."*

A large number of reported investigations is based upon a relatively small number of observations. In some cases, doubtless, deductions thus based will probably hold universally, in other cases they will not.

The investigators who have studied groups of 6,000 or more children are E. O. Lewis, J. M. Rice, C. W. Stone, S. A. Courtis, and Miss Rose A. C. Carrigan, the last named being officially assigned by the Boston School Committee to continue the extensive investigations of the Boston schools made by S. A. Courtis.

Rice's Test.—In 1902, Dr. J. M. Rice gave a test of eight problems to 6,000 children of grades IV to VIII in eighteen schools of various types in seven cities. He reached the conclusion that supervision was the most important factor in city systems, and the success of supervision was ultimately dependent upon the efficiency of the system of examinations employed. He was the first to show that results of school work in arithmetic were not proportional to the time given the subject on the school program.

Stone's Test.—In 1908, Dr. C. W. Stone published the results of six years' study of detailed data from arithmetic tests in twenty-six widely scattered school systems.†

He concludes: "The course of study may be the most important single factor, but it does not produce abilities unless taught. The other essential features for successful teaching are teachers and children of usual abilities, a reasonable time allotment, intelligent supervision and adequate measurement of results by tests."‡

Some other special conclusions of Dr. Stone are as follows:

The same system occupies a decidedly different rank in fundamental operations from that held in reasoning.

* Pedagogical Seminary, June, 1914, Vol. XXI., pp. 194, 195.

† "Arithmetical Abilities and Some Factors Determining Them," by C. W. Stone, Ph.D.

‡ *Ibid.*, p. 91.

The correlation of reasoning is lowest, 32 per cent., with addition; highest, 52 per cent., with subtraction.

Brown's Test.—In 1911 and subsequently, J. C. Brown's application of the Stone standard tests to 6th, 7th, and 8th grade children, showed the value of regular five-minute drills in the fundamentals; permanent improvement resulted in a marked degree also in text-book work.*

Courtis's Tests.—After experimenting with the Stone tests, S. A. Courtis, of Detroit, devised standard tests on a more carefully devised basis than any tests previously constructed. Basing his studies on more than nine thousand children, he was able to classify the combinations of two figures according to difficulty in each of the four fundamental operations. Thus he could construct standard scales of measurement in each of these operations and could duplicate them making several series of equivalent tests. In the preliminary work much difficulty was experienced in controlling individual variation from the accepted standards. The testing of 5,000 children in Detroit and 33,000 in New York for the Hanus Committee on School Inquiry made possible unusual uniformity in the conditions under which the tests were given and in tabulating the results, so that the data were both more extensive and more reliable than from any previous tests. Later the 20,000 children in the Boston schools were tested during the school year 1912–1913. These extensive investigations together with tests in nearly one hundred other school systems in some fifteen states, easily place these tests in the front rank of such investigations.

Mr. Courtis's conclusions from these tests are:

(*a*) That school work as now conducted is exceedingly inefficient.

(*b*) That results vary but little from school to school and from city to city.

(*c*) That the factors, course of study, method of work, ability of the teachers, are of relatively slight importance compared to the basic factor, the differences in the aptitudes and needs of individual children.

(*d*) That the problem of problems in education today (1914)

* *Journal of Educational Psychology,* February, 1911; November and December, 1912.

is the working out of administrative and pedagogic methods that will enable the teacher of a large class to give individual instruction to each particular child according to its needs.

To give some idea of the consistency and the uniformity of the results of these tests the following scores are given:

MEDIAN SCORES, SERIES B.

FEBRUARY, 1914, TABULATION.

Attempts. *Test 1—Addition.* Rights.

Source of Scores.	Attempts.				Rights.			
	Detroit.	Boston.	General.	Probable June Standards.	Detroit.	Boston.	General.	Probable June Standards.
No. in Group.	1,315.	20,441.	3,618.		1,315.	20,441.	3,618.	
Grade 3			3.6	4.0			.7	2.0
4	5.4	5.3	4.7	6.0	2.7	2.6	1.9	3.0
5	6.7	7.2	5.8	7.5	3.9	3.7	3.9	4.0
6	8.4	8.3	8.0	9.0	4.6	4.9	3.7	5.0
7	9.2	8.4	8.2	10.5	5.4	4.6	4.7	6.5
8	10.2	11.0	9.7	12.0	6.7	7.8	5.6	8.0
Test 2—Subtraction.								
Grade 3			3.8	4.0			9.7	1.0
4	5.6	5.5	5.7	6.0	3.1	2.5	1.2	3.0
5	8.0	7.6	6.5	8.0	5.5	5.6	4.5	5.5
6	8.8	8.8	8.9	10.0	6.2	6.3	6.1	7.0
7	9.8	9.1	10.2	11.5	7.3	6.9	7.8	8.5
8	12.3	11.4	11.7	12.5	9.5	8.6	8.4	10.0
Test 3—Multiplication.								
Grade 4	3.6	2.2	3.9	4.5	1.0	1.3	1.3	1.5
5	6.4	5.8	6.0	7.0	3.8	3.3	2.6	4.0
6	7.4	6.9	7.2	8.5	4.8	4.8	4.5	5.5
7	9.6	8.0	8.4	10.0	6.0	5.1	5.2	6.5
8	10.5	9.5	9.9	11.5	7.5	7.6	6.4	8.0
Test 4—Division.								
Grade 4	1.9	2.6	3.1	3.5	.7	.7	.7	1.0
5	4.9	4.5	4.5	5.0	2.7	2.0	2.3	3.0
6	6.4	5.8	5.8	6.5	4.4	3.3	4.3	5.0
7	8.6	6.1	7.6	8.5	7.1	5.1	5.8	7.0
8	10.3	8.8	9.2	10.5	8.8	6.9	6.3	9.0

(Manual of Instruction, The Courtis Standard Tests, page 75, Department of Co-operative Research, 82 Eliot Street, Detroit, Mich.)

The score in the above table of median scores is the ***number*** of examples attempted in the "Attempts"; and the ***number*** of examples right in the "Rights." A time limit was set in each test,—in no case over eight minutes. Each test contained sufficient examples to more than occupy the time of the swiftest computor.

To give an idea of the range of difficulty in these standard tests, a unit example from each is given below. The other examples are similar to the ones here given but with other combinations. (It must be borne in mind that the examples are carefully constructed according to the formulas and principles developed by the author, and are not chance combinations of figures.)

SAMPLE EXAMPLES FROM THE TESTS.

Test 1. Addition (Time 8 Min).	Test 2. Subtraction (Time 4 Min).	Test. 3. Multiplication (Time 6 Min).	Test 4. Division (Time 8 Min).
297	75088824	3597	94)85352
925	57406394	73	
473			
983			
315			
661			
794			
177			
124			

While Mr. Courtis has also devised other tests in combining the elementary operations, in copying figures, and in simple reasoning, the four tests of which the scores are given above, have had widest application and are the best basis from which to form deductions. It will be noticed that Mr. Courtis in these tests has wisely studied only the foundations of arithmetic ability. The simple tests in reasoning which he devised, led him to the conclusion that ability in reading was an important factor in solving arithmetical problems in simple reasoning.

Mr. Courtis's tests have clearly demonstrated the complexity of the problem facing every teacher of arithmetic. Though her class is officially designated, Fourth, Fifth, Sixth Grade, etc., she is really teaching pupils belonging (from the arithmetic point of view) to all grades, and to make matters worse, most of

the pupils belong in other grades than the one she is expected to teach. This tremendous flaw in our system of classification is the fundamental source of our ineffective results. In reading Mr. Courtis's conclusions based on these extensive tests (as previously quoted) we must bear in mind this lack of scientific classification, which so hampers the work of a trained and able teacher as to almost reduce her work to the level of the unskilled and indifferent worker. Space will not permit further consideration and criticism of many other of Mr. Courtis's observations. His aim to reach the individual child and his recognition of the wide range of aptitudes and needs of individual children are safe planks in any educational platform, whether it be of the teacher or that of the educational investigator.

Boston Tests Now in Progress.—Further experiments and investigations by means of the Courtis tests are now being conducted in the Boston schools by a special bureau assigned to the work under the direction of Miss Rose A. Carrigan. The results of these investigations are not yet ready for publication, but may be looked for in the forthcoming reports of Superintendent Dyer.

Jessup's Questionnaire.—In the *Elementary School Teacher*, Vol. 14, June, 1914, Prof. Walter Jessup presents the results of a questionnaire on topics to be taught and time to be given to them. This was sent to all cities with a population of 4,000 and over and to one sixth of the county superintendents of the United States.

The returns indicate a strong tendency to omit many of the traditional topics of arithmetic and to emphasize other phases of the subject that are of more immediate social and economic value. The report includes a list of twenty-five topics showing the percentage of superintendents who favor the elimination of each. A list of nineteen topics together with the percentage of superintendents who favor increased emphasis upon each of them gives suggestive and valuable evidence to superintendents, supervisors and teachers of arithmetic.

The report also states the median time spent on arithmetic in each of the grades. The author suggests that this median percentage of recitation time might profitably be adopted as a

standard until scientific investigations have proven it to be in error.

In the November number of the same magazine Prof. Jessup gives additional data, from these same questionnaires, bearing on the proper time of introduction of an arithmetic text. The returns show that while there is a good deal of variation in this respect, 56.1 per cent. of the schools reporting introduce a text in arithmetic in the third grade and 27.7 per cent. of the schools introduce a text in the fourth grade. In a very few schools the text is introduced as early as the first grade, while in some schools no text is used prior to the sixth grade. The greatest variations are found in the smaller cities.

Lewis's Experiment.—The experiment of E. O. Lewis (*Journal of Experimental Pedagogy,* Vol. 2, No. 2, June, 1913) to ascertain the comparative popularity of arithmetic is called by the author but a "first word" on the subject. Eight thousand pupils in the schools of London and South Wales were asked to write their school subjects in order of preference. Mr. Lewis makes his report in such a way that it is impossible to state the relative popularity of arithmetic from the individual viewpoint. The papers from each class examined were averaged, forming class preferences; the class preferences were then compared. Arithmetic appears to hold a middle ground in the esteem of the classes. The author points out the fallacy of applying this position to the individuals and states: "Arithmetic was generally either very high on the list or very low. Generally speaking, arithmetic is popular with a considerable section of the boys, especially in the classes; and disliked by a large section of the girls."

Suggestions and Recommendations.

1. Experimental pedagogy should be encouraged and teachers should investigate in a scientific manner some of the many questions that face our profession in teaching arithmetic.

2. Standards in most of our schools for speed and accuracy in the fundamental operations are now so low that much good will be accomplished by the testing method now generally used by Courtis and others which emphasizes these qualities and makes them the basis of many deductions as to the individual ability of the pupil and the pedagogical skill of the teacher.

3. In conformity with the experience of a great many teachers, experiments have clearly shown the value of continued brief, snappy drill to increase accuracy and speed and to maintain a creditable standard, provided the drill is intelligently directed toward the objects in view. We recommend that such drills form a definite part of the program of study.

4. While we feel that great improvement is desired in speed and accuracy in American schools, yet teachers must always keep in mind that mechanical skill in arithmetic is *but a part* of the complex training which good arithmetic teaching should give. Among the many other qualities desired, the cultivation of the power of analysis and logical deduction, the distinction between cause and effect, clearness and accuracy in thought and statement, should always be important objects for every teacher. Reasoning power is so complex that its measurement is exceedingly difficult. The large tests made in New York and in Boston show that reasoning power is independent of skill in the fundamental operations. A group of investigators in the fields of geometry or analysis would not compare in arithmetical accuracy and speed with the work of trained computors.

5. There are undoubtedly many valuable factors in the teaching of arithmetic which are not susceptible of distinct separation and numerical tabulation. Other factors, though probably measurable, are complex and involved. Hence it is desirable to examine the simplest mechanical processes first.

6. In accordance with the above suggestions, we recommend that individual teachers and school systems give tests which have had wide application and compare their results with the widely established standards, noting the agreements and differences. The latter should be explained if possible. The results, including explanations of anomalies and suggestions arising from the local test, should be communicated to the investigators whose standards were used.

7. Conclusions should not be drawn from a small number of observations, neither should statistics be selected to prove a preconceived idea. Questions arising from our experience may arouse fruitful investigations, but it is just as important to disprove an erroneous assumption as it is to confirm our suspicions. We feel that extended and unwarranted deductions from the

simple tests in the fundamentals of arithmetic will tend to defeat their primary purpose of improving school conditions and will create a strong opposition to them, largely based on the grounds which resulted in the overthrow of the severe examination system formerly prevalent in our schools.

8. In conclusion, we feel that the tests already given by Courtis and others have clearly detected grave technical weakness in many schools in the fundamentals of arithmetic, and that such carefully devised tests should be used for this purpose. When such weakness is detected, remedial measures should be instituted to avoid the complex situation proved by recent investigations to exist in New York, Boston, and elsewhere. Scientific methods and effective teaching are impossible when large classes comprise many grades of arithmetic ability in the fundamental processes. This is the situation now confronting our teachers and we urgently recommend this association to use its influence in correcting this glaring defect in the supervision and organization of our schools.

Respectfully submitted,

MAURICE J. BABB,
University of Pennsylvania, Philadelphia,
J. C. BROWN,
Teachers' College, New York,
ARTHUR W. CURTIS,
State Normal School, Oneonta, N. Y.,
CHARLES C. GROVE,
Columbia University, N. Y.,
EMMA WOLFENDEN,
William Penn High School, Philadelphia,
JONATHAN T. RORER, *Chairman,*
William Penn High School, Philadelphia,
Committee.

NEW BOOKS.

Live and Learn. By WASHINGTON GLADDEN. New York: The Macmillan Company. Pp. 159. $1.00 net.

We have here a collection of addresses directed for the most part to the young, but which have a large value for parents and teachers. The author's endeavor is to show that by learning how to see, how to hear, how to speak, how to think, how to give, how to serve, how to win and how to wait, the individual may make most of their opportunities and best develop their characters. The rich experience and ripe judgment of the author are well known and these addresses will be highly appreciated for their strength.

What Can I Know? By GEORGE TRUMBULL LADD. New York: Longmans, Green and Co. Pp. 311.

This title at once challenges our attention and we hasten to find an easy answer to the question and discover in the end how difficult it is. In twelve chapters whose headings are: Meaning of the question; What is it to know?; On thinking one's way through a subject; On being sure of what we know; Degrees and limits of knowledge; Agnostics and people of common-sense; Knowledge and reality; What is the use of knowing?; The value of the man who knows; Can a man know God?; there is much of interest to teachers and perhaps especially to teachers of mathematics who are apt to feel the certainty of some things that we are not so sure about.

Inductive versus Deductive Methods of Teaching. By W. H. WINCH. Baltimore: Warwick and York. Pp. 146. $1.25.

This is number eleven of the *Educational Psychology Monographs* and should prove of great interest to every teacher of mathematics and especially to those of geometry. In five different schools of London, attended by children of different social classes, a series of experiments were made to test the relative values of inductive and deductive methods of teaching as applied to geometrical definitions. It was found that in three of the schools tested those taught deductively could reproduce precisely what they had been taught better than those taught inductively. In the other two the results were about even. In testing to find out which of the two methods gave the better results when the children were tested on *new material* it was shown that in all five schools those taught inductively did better work. This would seem to be a strong argument in favor of the syllabus method of teaching geometry.

Miss Madelyn Mack, Detective. By HUGH C. WEIR. Boston: The Page Company. Pp. 328. $1.25 net.

A series of five detective stories, in which Miss Mack, the wonderful woman detective, is a regular Sherlock Holmes in unravelling the intricate mysteries set before her. The book is well illustrated and is sure to hold the interest of the reader as the clues are followed to the last page of the solution.

American Composers. By RUPERT HUGHES and ARTHUR ELSON. Boston: The Page Company. Pp. 582. $2.50 net.

This is a new and revised edition of this work on contemporary American composers and besides giving a rather comprehensive account of the music of the country it gives a biography of the leaders. It is the author's belief that some of the best music in the world is being written here at home and that it only needs to be brought to light to be appreciated. The book is illustrated with autographs and pictures of many of the composers.

Bulgaria and Her People. By WILL S. MONROE. Boston: The Page Company. Pp. 410. $3.00 net.

The author of this book was in Bulgaria during the last war and has given an authentic account of the geography, history, peoples, institutions, education, religion, commerce, art, music, literature, and leading cities of the country. He also gives an account of the Bulgars in Macedonia.

Professor Monroe considers that the Bulgars have outdistanced the other peoples of the Balkan peninsula in most of those matters that make for social progress and civilization. It is a timely and entertainingly written book and is profusely illustrated by special photographs.

The Spell of Japan. By ISABEL ANDERSON. Boston: The Page Company. Pp. 396. $2.50 net.

In this book the interesting country of Japan is portrayed by the author in a very pleasing way. Her experiences there as wife of the American ambassador gave her a splendid chance to learn a great deal about the life, the customs, the court life, the festivals, the shrines, the amusements and the literature of the Japanese people. She also saw much of the natural beauty of this Island Empire and has told it all in an entertaining manner. It is beautifully illustrated.

The Spell of Spain. By KEITH CLARK. Boston: The Page Company. Pp. 425. $2.50 net.

Mr. Clark has given a vivid description of the places of special interest in this country and has told the story so well that the reader feels under the "spell." There is not only a description of the country but of the people, their customs and their religion.

Like the other volumes of this "spell" series, it is beautifully illustrated with special pictures and photo reproductions. The publishers are to be congratulated on the excellence of the series.

Blue Bonnet in Boston. By CAROLINE E. JACOBS and LELA H. RICHARDS. Boston: The Page Company. Pp. —. $1.50 net.

A delightful story of a wealthy Texas girl who goes to an expensive boarding school in Boston. The book tells of her days there, giving an account of her school duties and studies as well as the social life and larks.

Our Knowledge of the External World as a Field of Scientific Method in Philosophy. By BERTRAND RUSSELL. Chicago: The Open Court Publishing Co. Pp. 245. $2.00.

In eight lectures the author considers: current tendencies; logic as the essence of philosophy; our knowledge of the external world; the world of physics and the world of sense; the theory of continuity; the problem of infinity considered historically; the positive theory of infinity; the notion of cause and its application to the free will problem. It is an endeavor to show by means of examples the nature, capacity and limitations of the logical-analytical (mathematical) method in philosophy. It considers the theories of the past as useful and worthy of study, but if philosophy is to become a science it must follow this new method.

The True Ulysses S. Grant. By CHARLES KING. Philadelphia: The J. B. Lippincott Company. Pp. 400. $2.00 net.

The writer of this volume came to his task with great interest and in the belief that there were virtues possessed by Grant which had not been given due prominence by earlier biographers. Being one of Grant's subalterns and having an intimate knowledge of his every day life, the author gives a clear and interesting pen picture of this great American. It will take its place among the "True Biographies" series as a valuable addition to the group and supplies many sidelights on the history of this important period.

Laird & Lee's Diary and Time-Saver maintains its high standard and as usual has something new. Besides the diary proper it contains information about the Panama-Pacific Exposition, the Panama-California Exposition at San Diego, the moon's phases for 1915, population of U. S. cities of 10,000 and up, Panama Canal statistics and map, calendars, first historical things, birthstones, state flowers, wages table, surgical and medical hints, business laws, legal weights per bushel, cash account pages, page for telephone numbers one uses, astrological signs for each day in the year. Attractive full flexible leather binding, gild edges, gold title, 25c. Press of Laird & Lee, Inc., Chicago.

NOTES AND NEWS.

THE TWELFTH ANNUAL MEETING OF THE ASSOCIATION OF MATHEMATICAL TEACHERS IN NEW ENGLAND.

The twelfth annual meeting of this association was held in the buildings of the Massachusetts Institute of Technology on December 5, 1914, at ten o'clock in the forenoon and at two o'clock in the afternoon.

The committee on nominations which consisted of Prof. E. V. Huntington, Mr. F. P. Morse, and Mr. A. V. Galbraith, reported the following candidates for the offices for 1915.

President, Professor J. L. Coolidge, Harvard University.

Vice-President, Professor F. C. Ferry, Williams College.

Treasurer, Mr. F. W. Gentleman, Mechanic Arts High School, Boston.

Members of the Council, Mr. Elmer Case, Brookline High School; Miss Harriet R. Pierce, Worcester High School.

To fill the vacany caused by the resignation of Mr. Edwin A. Shaw, Mr. William L. Vosburgh, Boston Normal School.

These candidates were elected.

The report of the treasurer for the year 1913–1914 was read, accepted and placed on file. The committee on Pre-High School Mathematics consisting of Mr. William L. Vosburgh, chairman, Mr. A. Lawrence Murphy, Miss Amelia A. Hall, Mr. A. Harry Wheeler, made a preliminary report of progress.

The morning program consisted of four short papers on the detail of mathematical instruction in the secondary school. These topics brought forth some very interesting and profitable discussion. The large attendance of about a hundred and fifty gave evidence that the general character of the topics and discussion was of the type which our members find profitable.

The luncheon at the Technology Union was attended by about sixty, and formed a very enjoyable social feature of the day's program.

The first speaker of the afternoon session was Professor Birkhoff, of Harvard University, who presented to the associa-

tion a "Simple Unsolved Mathematical Problem" which should suggest to the teachers of secondary mathematics an interesting topic for study. The problem of coloring a map with not more than four colors, which was Professor Birkhoff's topic, was of a nature simple enough to be within the reach of those of us who are not keeping our higher mathematics fresh in our minds by the teaching of advanced courses; while, at the same time it was seen to be difficult enough to call for the serious study which would make its further investigation a stimulating piece of work for those who wish to broaden their outlook by dipping into subjects outside the details of the topic which they happen to be teaching. The substance of this address is given in an article by Prof. Birkhoff in the American Journal of Mathematics, Vol. XXXV, No. 2, pp. 115–128.

The preliminary report of the Committee on the Present Status and Welfare of Mathematics in the Secondary Schools which consists of Professor H. W. Tyler, chairman, Professor F. C. Ferry, Professor Sarah E. Smith, Miss Harriet R. Pierce, Mr. George W. Evans, Mr. A. V. Galbraith, was read by the chairman. This report, although it was only a preliminary one, gave evidence of the thoroughness and faithfulness with which the committee is doing this important piece of work. This report also served as an introduction to the address by Mr. Henry C. Morrison, State Superintendent of Public Instruction in New Hampshire. Mr. Morrison's paper will be given in a later issue of the MATHEMATICS TEACHER.

This address was followed by a general discussion of the questions involved in the report of the above committee and Mr. Morrison's paper. Professor Ferry opened the discussion in which Professors Coolidge and Huntington and Dr. Snedden joined.

The committee has had extended correspondence with Commissioner Snedden which is likely to be published in the near future.

The committee has recently organized sub-committees on The Status of Algebra; The Practicability of an Introductory, Composite Course; The Training of Teachers of Mathematics; School Programs; Psychological Aspects of the Subject; Mathematics for Girls.

A syllabus for each topic is in preparation, and correspondence with persons interested would be welcome.

New Members.

Adams, Alfred S., 22 Vine St., Auburn, Me.
Beatley, Ralph, Milton Academy, Milton, Mass.
Hale, Arthur W., 43 Newell Road, Auburndale, Mass.
Hoyt, Miss Amelia A., 3 Robinson Ave., Danbury, Conn.,
Merritt, H. T., Brewster Free Academy, Wolfeboro, N. H.
Robinson, Miss Fannie H., 142 Hammond St., Bangor, Me.
Waterman, Paul W., Volkman School, Boston, Mass.

New Members Added Since November 1, 1914.

Akers, Winfred C., 108 University Road, Brookline, Mass.
Bates, F. Jay, Barton Academy, Barton, Vt.
Brayton, Percy F., 136 Milton St., West Medford, Mass.
Birkhoff, Prof. George D., 49 Shepard St., Cambridge, Mass.

Bush, Vannevar, Tufts College, Mass.
Chaffie, Thomas C., 9 Elm St., Gardiner, Me.
Childs, Earle A., High School, Simsbury, Conn.
Clow, Arlington I., West Warwick High School, R. I.
Cook, Miss M. Imogene, Walnut Hill School, Natick, Mass.
Dunn, Joseph B., Mechanic Arts High School, Boston, Mass.
Eaton, Guy H., Phillips Academy, Andover, Mass.
Fash, Frank E., Box 227, Millis, Mass.
Gilbert, William H., High School, Gilbertsville, Mass.
Hayes, Miss Bernice M., Hillsboro, N. H.
Moulton, Miss Olive H., High School, Sanford, Me.
Peaslee, Arthur N., St. George's School, Newport, R. I.
Poor, Miss Helen W., 22 Yale Ave., Wakefield, Mass.
Roche, John C., English High School, Boston, Mass.
Smith, William E., Supt. of Schools, Fairfield, Conn.
Swift, Prof. Elijah, 433 South Willard St., Burlington, Vt.
Wellman, Justin O., Colby Academy, New London, N. H.

The spring meeting of the Association of Teachers of Mathematics for the Middle States and Maryland will be held at Hunter College, New York City, on April 17.